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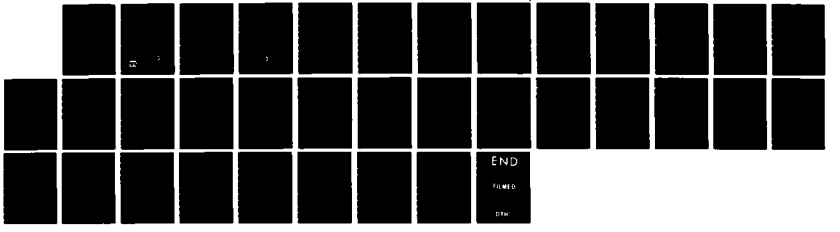

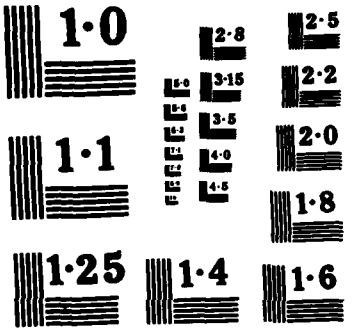

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BUG DISTRIBUTION AND PATTERN CLASSIFICATION

KIKUMI K. TATSUOKA

MAURICE M. TATSUOKA

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A model (called rule space) which permits measuring cognitive skill acquisition, diagnosing cognitive errors, detecting the weaknesses and strengths of knowledge possessed by individuals was introduced earlier. This study further discusses the theoretical foundation of the model by introducing "Bug Distribution" and hypothesis testing (Bayes' decision rules for minimum errors) for classifying an individual into his/her most plausible latent state of knowledge. The model is illustrated with the domain of fraction arithmetic and compared with the results obtained from a conventional Artificial Intelligence approach.

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Introduction

Several deterministic methods commonly used in Artifical Intelligence have been applied to develop problem-solving programs, or error-diagnostic systems. These methods have successfully diagnosed many erroneous rules of operation in arithmetic, algebra, and some science domains. The results of such error analyses have contributed to our current understanding of human thinking and reasoning.

These approaches, however, lack taking the variability of response errors into account, and they also depend on a specific model of problem solving. Therefore, they often cannot diagnose responses affected by random errors (sometimes called "slips") or produced by innovative thinking that is not taken into account by the current models. It is very difficult to develop a computer program whose underlying algorithms for solving a problem represents a wide range of individual differences. Yet, when these diagnostic systems are used in educational practice, they must be capable of evaluating any responses on test-items, inconsistent performances as well as those yielded by creative thinking. Therefore, we need a model that is capable of diagnosing non-systematic cognitive errors and is also capable of evaluating non-conventional problem-solving activities.

Tatsuoka and her associates (Tatsuoka, 1985, 1984a; Tatsuoka & Linn, 1983; Tatsuoka & Tatsuoka, 1983, 1982) have developed such a model called rule space and have successfully applied it to diagnose misconceptions possessed by students in signed-number and fraction arithmetic. The model maps all response patterns into a set of ordered pairs, the latent ability variable θ and one of the IRT based caution indices (ζ) introduced by Tatsuoka (1984a). However, the approach used in their model lacks, somehow, a sound statistical foundation in expressing

random errors when a specific rule is applied for solving a problem. The simulation study by Tatsuoka and Baillie (1982) showed that the response patterns yielded by not-perfect-applications of a specific erroneous rule of operation in a procedural domain form a cluster around the rule. Moreover, they found empirically that the two random variables, θ and ζ obtained from those response patterns in the cluster follow a multivariate normal distribution. This cluster around a rule is called a "bug distribution" hereafter. The theoretical foundation of this empirical evidence will be discussed in this study. First, a brief description of the probabilistic model introduced in Tatsuoka (1985) will be given. Then the connection of each "bug distribution" to the model will be discussed in the conjunction with the theory of statistical pattern classification and recognition.

Distribution of Responses around an Erroneous Rule

The responses around a particular rule of operation in a procedural domain which are produced by not-perfectly-consistent applications of the rule to the test items form a cluster. They include responses which deviate, in various degrees of remoteness, from the response generated by the rule. When these discrepancies are observed, they are considered as response errors. These response errors are called "slips" by cognitive scientists (Brown & VanLehn, 1980). The properties of such responses around a given erroneous rule will be investigated in this section.

First, the probability of having a "slip" on item j (j=1,2,...,n) is assumed to be the same value, p for all items and it will be called "slip probability" in this paper. Let us denote an arbitrary rule for which the total score is r by Rule R and let the corresponding response pattern be:

(1) $$R = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_r \\ x_{r+1} \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \quad x_1 = x_2 \ \ldots = x_r = 1, \text{ and } x_{r+1} = \ldots = x_n = 0.$$

The response patterns existing one slip away from Rule R are of two kinds: a slip of "one to zero" occurring at $1 \leq j \leq r$ and "zero to one" at $r < j \leq n$. The number of response patterns having one slip is therefore $\binom{r}{1}\binom{n-r}{0} + \binom{r}{0}\binom{n-r}{1}$, and the probability of having one slip on items $j=1,\ldots,n$ is given by $\binom{r}{1} p^1 (1-p)^{r-1} \binom{n-r}{0} p^0 (1-p)^{n-r} + \binom{r}{0} p^0 (1-p)^r \binom{n-r}{1} p^1 (1-p)^{n-r-1}$ if the probability p is the same for all items, $j=1,\ldots,n$. Therefore the following equation (2) is obtained;

(2) Prob (x_j - 1 for some $j=1,\ldots,r$ or x_j + 1 for some $j=r+1,\ldots,n$) = Prob (having a slip on an item) = $\left(\binom{r}{1}\binom{n-r}{0} + \binom{r}{0}\binom{n-r}{1}\right) p^1(1-p)^{n-1}$.

Similarly, the probability of having two slips on the items is given by Equation (3) as follows:

(3) Prob (having two slips on the items) = $\left(\binom{r}{2}\binom{n-r}{0} + \binom{r}{1}\binom{n-r}{1} + \binom{r}{0}\binom{n-r}{2}\right) p^2 (1-p)^{n-2}$.

In general, the probability of having k slips on the items is given by:

(4) Prob (having k slips on the items)

$$= \left(\sum_{k_1+k_2=k} \binom{r}{k_1}\binom{n-r}{k_2} \right) p^k (1-p)^{n-k} .$$

The generating function of the distribution of frequencies up to k slips will be given by Equation (5) as follows:

$$(5) \quad \sum_{s \le k} \text{Prob (having s slips)} = \sum_{s \le k} \left(\sum_{s_1+s_2=s} \binom{r}{s_1} \binom{n-r}{s_2} \right) p^s (1-p)^{n-s} .$$

Since the coefficient term inside the braces equals $\binom{n}{s}$, Equation (5) will be simply a binomial distribution, given by Equation (6).

$$(6) \quad \sum_{s \le k} \text{Prob (having s slips)} = \sum_{s \le k} \binom{n}{s} p^s (1-p)^{n-s} .$$

Therefore, a cluster around Rule R which consists of response patterns including various numbers of slips (not-perfectly-consistent application of Rule R) has a frequency distribution of a binomial form with the equal slip probability p for the items. One weakness in Equation (6) is that the value of p is not known and it is very unlikely that the value of p is constant over the test items. If we assume each item has an unique slip probability, then the binomial distribution expressed by Equation (6) will be a compound binomial distribution. Equation (7) is the generating function of the compound binomial distribution.

$$(7) \quad \sum_{s \le k} \text{prob (having s slips)} = \sum_{s \le k} \prod_{j}^{k} (p_j + q_j)$$

Before an approximation of the slip probabilities p_j is discussed, the rule-space concept will be briefly introduced in the next section.

A Brief Summary of the Probabilistic model Rule Space

One of the purposes of the model, the rule space, is to interpret semantically the relationships among various erroneous rules and the right rule, and compare the characteristic of each rule to the right rule or other rules. An analogy for the underlying motivation of seeking a norm-referenced characteristic of "bug behavior" may be found in the theory and practice of norm-referenced tests. This starts by selecting the right rule as a norm and then comparing the other erroneous rules to the characteristic of the norm. By doing so, the psychometric behavior of "bugs" as compared with the right rule, understanding why and how various misconceptions are related and transformed from one to another will be explained more clearly than by just describing the list of bugs.

The rule space model begins by mapping all possible binary response patterns into a vector space of $((\theta, \zeta))$, where θ is the latent ability variable in Item Response Theory (IRT) and ζ (or $\zeta(\underset{\sim}{x};\theta)$) is one of the IRT based caution indices (Tatsuoka, 1984a; Tatsuoka & Linn, 1983). The mapping function $f(\underset{\sim}{x})$ is expressed as an inner product of two residual vectors, $\underset{\sim}{P}(\theta) - \underset{\sim}{x}$ and $\underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)$ where $P_j(\theta)$, $j=1,\ldots,n$ are the one- or two-parameter logistic-model probabilities, $\underset{\sim}{x}$ is a binary response vector and $T(\underset{\sim}{\theta})$ is the mean vector of the logistic probabilities. $f(\underset{\sim}{x})$ is a linear mapping function between $\underset{\sim}{x}$ and ζ at a given level of θ, and the response patterns having the same sufficient statistics for the maximum likelihood estimate $\hat{\theta}$ of θ are dispersed into different locations on the line of $\theta = \hat{\theta}$. For example, on a 100-item test, there are 4950 different response patterns

having the total score of 2. The ζ's for the 4950 binary patterns will be distributed between ζ_{min} and ζ_{max}, where ζ_{min} is obtained from the pattern having 1 for the two easiest items and zeros elsewhere, and ζ_{max} is from the pattern having 1 for the two most difficult items. $f(\underset{\sim}{x})$ has the expectation zero and variance $\sum_{j=1}^{n} P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2$ (Tatsuoka, 1985). Since the expectation of the random variable $x_j(j=1.,,,.n)$ is $P_j(\theta)$, the expectation of a vector $\underset{\sim}{x}$ is $\underset{\sim}{P}(\theta)$ whose jth component is $P_j(\theta)$. The vector $\underset{\sim}{P}(\theta)$ will be mapped to zero as shown in (8), thus the pattern corresponds to (0,0) in the rule space.

(8) $f(\underset{\sim}{P}(\theta)) = 0$

As for an erroneous rule R, the response vector $\underset{\sim}{R}$ given by (1) will be mapped onto $(\theta_R, f(\underset{\sim}{R},\theta_R))$, where the ζ value is $\sum_{j=1}^{n}(P_j(\theta) - R_j)(P_j(\theta) - T(\theta))$, and is given by (9). That is,

(9) $f(\underset{\sim}{R}) = -\sum_{j=1}^{r} Q_j(\theta_R)(P_j(\theta_R) - T(\theta_R) + \sum_{j=r+1}^{n} P_j(\theta_R)(P_j(\theta_R) - T(\theta_R))$.

Similarly, all the response vectors resulting from several slips around rule R will be mapped into the vicinity of $(\theta_R, f(\underset{\sim}{R}))$ in the rule space and form a cluster (called the cluster around R hereafter). Figure 1 shows computer-simulated examples of such clusters done on the PLATO system.

Insert Figure 1 about here

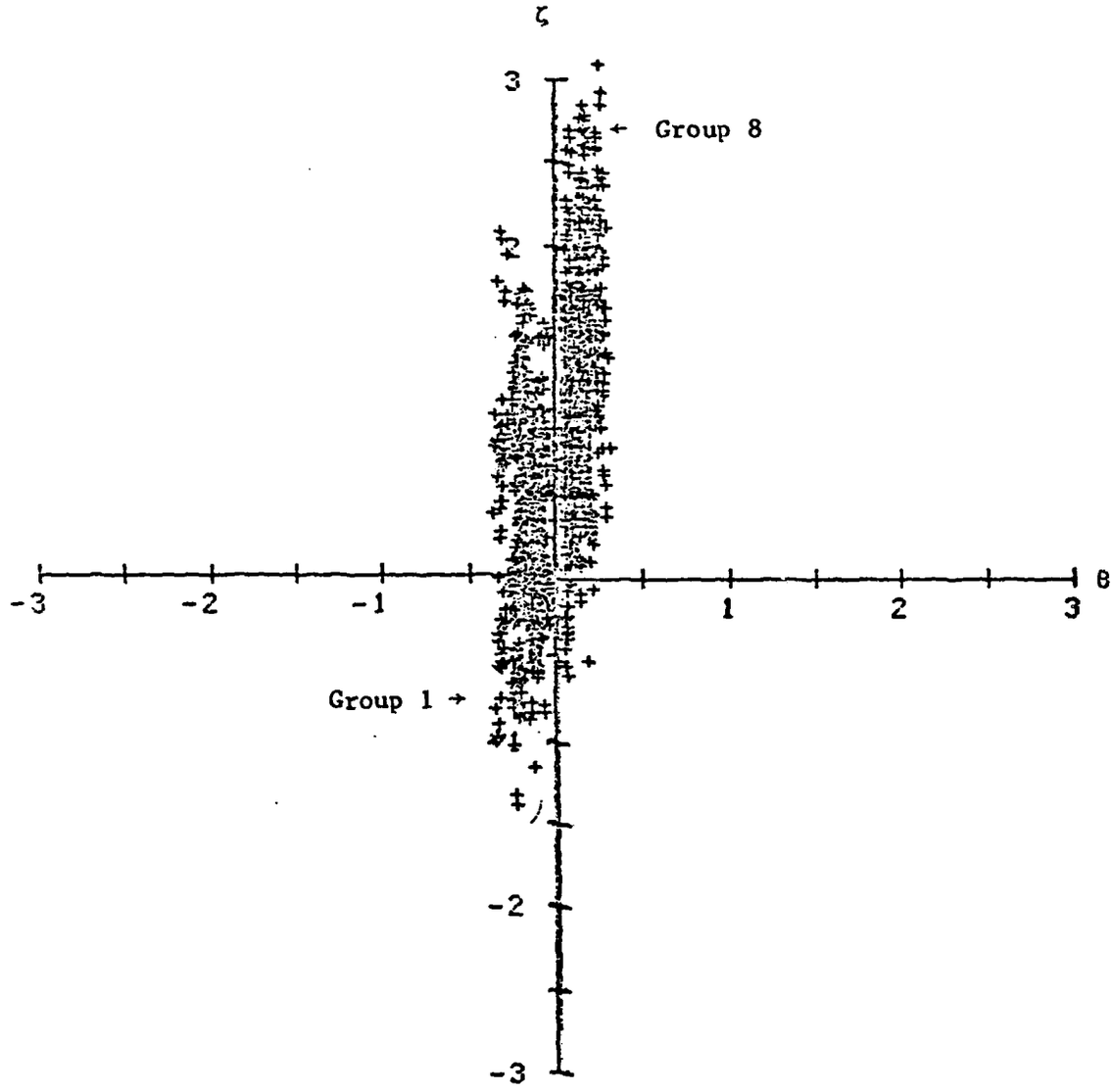

Figure 1: Two Clusters of Groups 1 and 8 with Two Slips. n=631

The two variables θ and $f(\underset{\sim}{x})$ are mutually uncorrelated so their covariance matrix has a diagonal form as follows;

$$(10)\quad \begin{bmatrix} \mathrm{var}(\hat{\theta}) & 0 \\ 0 & \mathrm{var}(f(\underset{\sim}{x})) \end{bmatrix} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & \Sigma P_j(\hat{\theta})Q_j(\hat{\theta})(P_j(\hat{\theta}) - T(\hat{\theta}))^2 \end{bmatrix}$$

where $I(\theta)$ is the information function of the test and is given by $\Sigma a_j{}^2 P_j(\theta)Q_j(\theta)$ where the a_j $(j=1,\ldots,n)$ are item discriminating powers.

Let us map all response patterns of the test, including clusters around various rules into the Cartesian product space of $\hat{\theta}$ and $f(\underset{\sim}{x})$, where

$$(11)\quad f(\underset{\sim}{x}) = (\underset{\sim}{P}(\theta), \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta)) - (\underset{\sim}{x}, \underset{\sim}{P}(\theta) - \underset{\sim}{T}(\theta))$$

$$\text{or} \qquad = K(\theta) - \sum_{j=1}^{n} x_j(P_j(\theta) - T(\theta)),$$

In particular, Rule R itself will be mapped as

$$\underset{\sim}{R} = \underset{\sim}{x} \rightarrow (\hat{\theta}_R, f(\underset{\sim}{R})),$$

where $f(\underset{\sim}{R})$ is given by Equation (9). The variance of the cluster around R will be expressed by using the slip probability of item j, p_j as follows;

$$(12)\quad \mathrm{Var}(\text{the cluster around R}) = \Sigma\, p_j q_j (P_j(\theta_R) - T(\theta_R))^2.$$

The quantities p_j and q_j are associated with Rule R as well as with item j, and their values are unknown. However, if the ordered pair (θ_R, ζ_R) in the rule space falls close to the θ axis, then p_j and q_j may be approximated by the logistic probability $P_j(\theta_R)$ and its complement $Q_j(\theta_R) = 1 - P_j(\theta_R)$,

respectively, without too much loss of accuracy. If p_j and q_j are thus approximated, then the variance of Equation (12) will be the same as the variance of the mapping function $f(x)$; that is

$$(13) \quad \text{Var}(\zeta \text{ in the cluster around } R) \simeq \Sigma\, P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2$$

The variance of θ in any cluster, on the other hand, is given by the reciprocal $1/I(\theta)$ of the information function, which can be computed as

$$(14) \quad \text{Var}(\theta \text{ in the cluster around } R) = 1/I(\theta_R)$$
$$= 1/\Sigma\, a^2_j\, P_j(\theta_R)Q_j(\theta_R)$$
where $a_j = 1$ for the one-parameter logistic model.

The above two variances, along with the fact that ζ and $\hat{\theta}$ are uncorrelated, plus the reasonable assumption that they have a bivariate normal distribution, allow us to construct any desired percent ellipse around each rule point $\underset{\sim}{R}$. The upshot is that, if all erroneous rules (and the correct one) were to be mapped into the rule space along with their neighboring response patterns representing random slips from them, the resulting topography would be something like what is seen in Figure 2. That is, the population of points would exhibit modal densities at many rule points that each forms the center of an enveloping ellipse with the density of points getting rarer as we depart farther from the center in any direction. Furthermore, the major and minor axes of these

---

Insert Figure 2 about here

---

ellipses would -- by virtue of the uncorrelatedness of ζ and $\hat{\theta}$ -- be parallel to the vertical (ζ) and horizontal ($\hat{\theta}$) reference axes of the rule space, respectively.

Recalling that for any given percentage ellipse, the lengths of the major and minor diameters are fixed multiples of the respective standard deviations

$$[\sum_{j=1}^{n} P_j(\theta)Q_j(\theta)(P_j(\theta) - T(\theta))^2]^{1/2} \text{ and } I(\hat{\theta})^{-1/2}$$

we may assert that the set of ellipses gives a complete characterization of the rule space. By this is meant that, once these ellipses are given, any response-pattern point can be classified as most likely being a random slip from one or another of the erroneous rules (or the correct one). We have only to determine, for a <u>suitable</u> percent value, which one of the several ellipses uniquely includes the given point.

<u>Operational Classification Scheme</u>

The geometrics scheme outlined above for classifying any given response-pattern point as being a "perturbation" from one or another of the rule points has a certain intuitive appeal (especially to those with high spatial ability!). However, it is obviously difficult if not infeasible to put it into practice. We, therefore, now describe the algebraic equivalent of the foregoing geometric clasification-decision rule, which is none other than the well-known minimum-D^2 rule, where D^2 is Mahalanobis' generalized squared-distance (Fukunaga, 1972; Tatsuoka, 1971). Then the

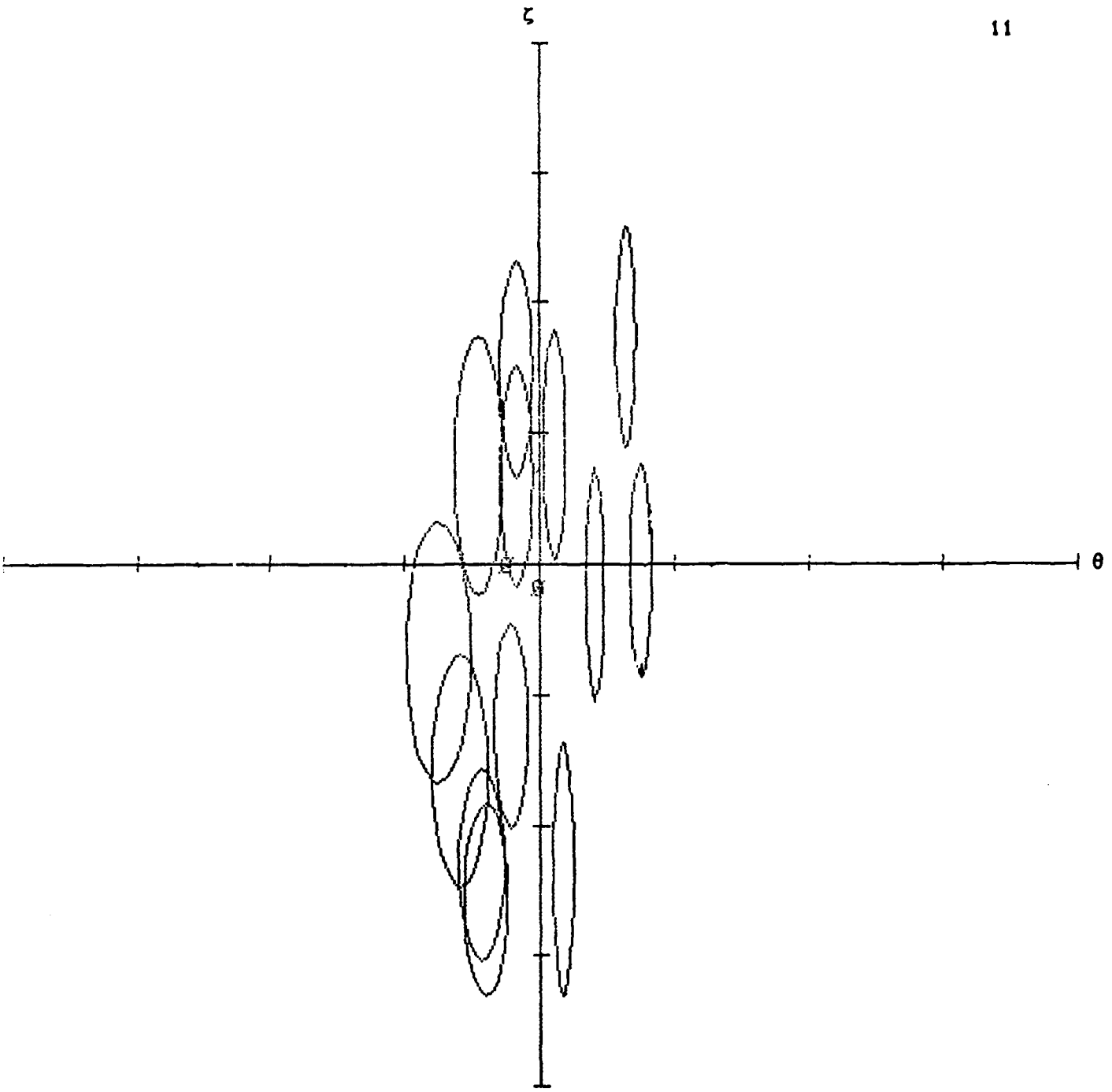

Figure 2: Fifteen Ellipses Representing Fifteen Error Types Randomly Chosen From Forty Sets of Ellipses

Bayes' decision rule for minimum error will be introduced.

Without loss of generality, we may suppose that a given response-pattern point $\underset{\sim}{x}$ has to be classified as representing a random slip from one of two rule points $\underset{\sim}{R}_1$ and $\underset{\sim}{R}_2$. Let $\underset{\sim}{Y}$ be a point in the rule space corresponding to $\underset{\sim}{x}$, $\underset{\sim}{X} = \begin{bmatrix} \hat{\theta}_x \\ f(\underset{\sim}{x}) \end{bmatrix}$. The Mahalanobis distance of $\underset{\sim}{X}$ from each of the two rule points is

$$(15) \quad D^2_{xj} = [\, \underset{\sim}{X} - \underset{\sim}{R}_j \,]' \, \underset{\sim}{\Sigma}^{-1} \, [\, \underset{\sim}{X} - \underset{\sim}{R}_j \,] \quad (j=1,2)$$

where $\underset{\sim}{R}_1 = \begin{bmatrix} \hat{\theta}_{R_1} \\ f(\underset{\sim}{R}_1) \end{bmatrix}$ and $\underset{\sim}{R}_2 = \begin{bmatrix} \hat{\theta}_{R_1} \\ f(\underset{\sim}{R}_2) \end{bmatrix}$, and the variance-covariance matrix will be,

$$\underset{\sim}{\Sigma} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & \mathrm{var}(f(\underset{\sim}{x})) \end{bmatrix}$$

The decision rule is, of course, to classify $\underset{\sim}{x}$ as a perturbation from $\underset{\sim}{R}_1$ if $D^2_{x1} < D^2_{x2}$ and otherwise as a perturbation from $\underset{\sim}{R}_2$. However, the decision based on the Mahalanobis distances, $D^2_{x_1}$ and $D^2_{x_2}$ does not provide error probabilities of misclassificaton. The next section will discuss them.

The Bayes Decision Rule for Minimum Error

Suppose R_1 and R_2 are two clusters of points corresponding to Rules 1 and 2, respectively. Let $\underset{\sim}{Y}$ be a vector $(\hat{\theta}, \xi)$ corresponding to an observed response pattern $\underset{\sim}{x}$, and ξ be the standardized value of $f(\underset{\sim}{x})$, IRT-based caution index. Then the variance-covariance matrix Σ will be

$$\underset{\sim}{\Sigma} = \begin{bmatrix} 1/I(\hat{\theta}) & 0 \\ 0 & 1 \end{bmatrix}$$

A decision rule based on probabilities may be summarized as follows:

(16) If $\text{Prob}(R_1 \mid \underset{\sim}{Y}) > \text{Prob}(R_2 \mid \underset{\sim}{Y})$ then $\underset{\sim}{Y} \in R_1$ and

if $\text{Prob}(R_1 \mid \underset{\sim}{Y}) < \text{Prob}(R_2 \mid \underset{\sim}{Y})$ then $\underset{\sim}{Y} \in R_2$.

These posterior probabilities can be obtained from prior probabilities, $\text{Prob}(R_1)$ and $\text{Prob}(R_2)$, and the conditional density function $p(\underset{\sim}{Y} \mid R_i)$, $i=1,2$ as follows:

$$(17) \qquad \text{Prob}(R_i \mid \underset{\sim}{Y}) = \frac{p(\underset{\sim}{Y} \mid R_i)\ \text{Prob}(R_i)}{p(\underset{\sim}{Y})}$$

Therefore, the decision rule can be expressed as follows:

(18) If $p(\underset{\sim}{Y} \mid R_1)\text{Prob}(R_1) > p(\underset{\sim}{Y} \mid R_2)\text{Prob}(R_2)$ then $\underset{\sim}{Y} \in R_1$

Otherwise, $\underset{\sim}{Y} \in R_2$.

This rule will be rewritten by using the likelihood ratio $\ell(\underset{\sim}{Y})$,

(19) If $$\ell(Y) = \frac{p(\underset{\sim}{Y} \mid R_1)}{p(\underset{\sim}{Y} \mid R_2)} > \frac{\text{Prob}(R_2)}{\text{Prob}(R_1)}$$, then $\underset{\sim}{Y} \in R_1$.

Otherwise, $\underset{\sim}{Y} \in R_2$.

Sometimes, it is convenient to take the negative log of the likelihood ratio in Expression (19), and rewrite it as Expression (20).

(20) If $h(\underset{\sim}{Y}) = -\ln \ell(\underset{\sim}{Y}) = -\ln(p(\underset{\sim}{Y} \mid R_1)) + \ln(p(\underset{\sim}{Y} \mid R_2))$

$< \ln [\ \text{Prob}(R_1) / \text{Prob}(R_2)\ ]$ then $\underset{\sim}{Y}$ belongs to R_1 .

However, the decision rule (20) does not lead to a perfect classification. As Overall (1972) states (p. 330)

> "Statistical classification decisions, like clinical diagnostic decisions, are only probabilistically correct. The clinician realizes this when he lists a secondary diagnosis. The statistician recognizes it more explicitly when he is able to assign a probability estimate to each classfication alternative."

The probability of error is the probability of $\underset{\sim}{Y}$ to be assigned to the wrong group, R_i.

Let us denote the Posterior density function by $P(R_i \mid \underset{\sim}{Y})$, prior density function of R_i by $P(R_i)$ and let Γ_1 and Γ_2 be the regions such that if $\underset{\sim}{Y} \in \Gamma_1$ then $P(R_1 \mid \underset{\sim}{Y}) > P(R_2 \mid \underset{\sim}{Y})$ and
if $\underset{\sim}{Y} \in \Gamma_2$ then $P(R_1 \mid \underset{\sim}{Y}) < P(R_2 \mid \underset{\sim}{Y})$.

The probability of error is given by the following equation:

$$(21)\quad \varepsilon = \text{Prob}(\underset{\sim}{Y} \in \Gamma_2 \mid R_1)\, P(R_1) + \text{Prob}(\underset{\sim}{Y} \in \Gamma_1 \mid R_2)\, P(R_2) .$$

Let us denote the probability of $\underset{\sim}{Y}$ belonging to Γ_2 when $\underset{\sim}{Y}$ is from R_1 by ε_1, then

$$\varepsilon_1 = \text{Prob}(\underset{\sim}{Y} \in \Gamma_2 \mid R_1) = \int_{\Gamma_2} p(\underset{\sim}{Y} \mid R_1)\, d\underset{\sim}{Y},$$

Similarly, the probability of $\underset{\sim}{Y}$ belonging to Γ_1 when $\underset{\sim}{Y}$ is from R_2, ε_2 will be

$$\varepsilon_2 = \text{Prob}(Y \in \Gamma_1 \mid R_2) = \int_{\Gamma_1} p(\underset{\sim}{Y} \mid R_2)\, d\underset{\sim}{Y} .$$

Then expression (21) can be rewritten by $\varepsilon = \varepsilon_1 P(R_1) + \varepsilon_2 P(R_2)$, or more precisely

$$(22)\quad \varepsilon = P(R_1) \int_{\Gamma_2} p(\underset{\sim}{Y} \mid R_1)\, d\underset{\sim}{Y} + P(R_2) \int_{\Gamma_1} p(\underset{\sim}{Y} \mid R_2)\, d\underset{\sim}{Y} .$$

That is, the total probability of errors is a weighted sum of the misclassification of samples from R_1 and R_2 into R_2 and R_1, respectively.

The integration of the conditional density function is necessary to get the error probability ε. The dimensionality of the conditional density function is often more than one, while the density function $p(\ell \mid R_i)$ of the likelihood ratio is one dimensional, so it is sometimes convenient to integrate the latter (Fukunaga, 1972). Hence, Equations (23) and (24) are used to obtain the error probabilities, ε_1 and ε_2:

$$\varepsilon_1 = \int_0^{P(R_2)/P(R_1)} p(\ell|R_1)d\ell \tag{23}$$

$$\varepsilon_2 = \int_{P(R_2)/P(R_1)}^{\infty} p(\ell|R_2)d\ell \tag{24}$$

If the density function $p(\underset{\sim}{Y} \mid R_i)$ is normal with expectations $\underset{\sim}{M}_i$ and covariance matrices $\underset{\sim}{\Sigma}_i$, the decision rule is summarized by the following statements:

(25) If $h(\underset{\sim}{Y}) = -\ln \ell(\underset{\sim}{Y})$

$$= \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_1)' \underset{\sim}{\Sigma}_1^{-1}(\underset{\sim}{Y} - \underset{\sim}{M}_1) - \tfrac{1}{2}(\underset{\sim}{Y} - \underset{\sim}{M}_2)' \underset{\sim}{\Sigma}_2^{-1}(\underset{\sim}{Y} - \underset{\sim}{M}_2)$$

$$+ \tfrac{1}{2} \ln \frac{|\Sigma_1|}{|\Sigma_2|} \begin{matrix} < \\ > \end{matrix} \ln \frac{P(R_1)}{P(R_2)} + Y \quad \begin{matrix} \in R_1 \\ \in R_2 \end{matrix} .$$

If $\underset{\sim}{\Sigma}_1 = \underset{\sim}{\Sigma}_2 = \underset{\sim}{\Sigma}$, then $h(\underset{\sim}{Y})$ becomes a linear function of $\underset{\sim}{Y}$ and the decision rule has the following form if $\underset{\sim}{Y}$ follows a normal distribution:

$$(26)\quad h(\underset{\sim}{Y}) = \tfrac{1}{2}\,(\underset{\sim}{Y} - \underset{\sim}{M}_1)'\,\underset{\sim}{\Sigma}^{-1}\,(\underset{\sim}{Y}-\underset{\sim}{M}_1) - \tfrac{1}{2}\,(\underset{\sim}{Y} - \underset{\sim}{M}_2)'\,\underset{\sim}{\Sigma}^{-1}\,(\underset{\sim}{Y} - \underset{\sim}{M}_2)$$

$$= \tfrac{1}{2}\,((\underset{\sim}{M}_2' - \underset{\sim}{M}_1')\underset{\sim}{\Sigma}^{-1}\underset{\sim}{Y} - \underset{\sim}{Y}'\underset{\sim}{\Sigma}^{-1}(\underset{\sim}{M}_1 - \underset{\sim}{M}_2) + \underset{\sim}{M}_1'\underset{\sim}{\Sigma}^{-1}\,\underset{\sim}{M}_1 - \underset{\sim}{M}_2'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}$$

$$= (\underset{\sim}{M}_2' - \underset{\sim}{M}_1')\underset{\sim}{\Sigma}^{-1}\underset{\sim}{Y} + \tfrac{1}{2}(\underset{\sim}{M}_1'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}_1 - \underset{\sim}{M}_2'\underset{\sim}{\Sigma}^{-1}\underset{\sim}{M}_2) \lessgtr \ln[P(R_1)/P(R_2)] = t.$$

then, $\quad Y \in \begin{cases} R_1 \\ \\ R_2 \end{cases}$

The error probability ε_1 is given by,

$$(27)\quad \varepsilon_1 = \int_t^{\infty} p(h(Y) \mid R_1)\,dh(\underset{\sim}{Y}) = \int_{\frac{t+\eta}{\sigma}}^{\infty} 1/\sqrt{2\pi}\ \exp\left(-\frac{Z^2}{2}\right) dZ$$

$$= 1 - \Psi\left(\frac{t+\eta}{\sigma}\right) .$$

where $t = \ln [p(R_1) / p(R_2)]$ and $\Psi(.)$ is the unit normal distribution. The conditional expectation of the likelihood function $h(\underset{\sim}{Y})$ is given by (28) and (29),

$$(28)\quad E(h(\underset{\sim}{Y}) \mid R_1) = -\tfrac{1}{2}\,(\underset{\sim}{M}_2 - \underset{\sim}{M}_1)'\,\underset{\sim}{\Sigma}^{-1}\,(\underset{\sim}{M}_2 - \underset{\sim}{M}_1) = -\eta$$

$$(29)\quad E(h(\underset{\sim}{Y}) \mid R_2 = +\tfrac{1}{2}\,(\underset{\sim}{M}_2 - \underset{\sim}{M}_1)'\,\underset{\sim}{\Sigma}^{-1}\ \ (\underset{\sim}{M}_2 - \underset{\sim}{M}_1) = +\eta$$

and, the variance of h(Y) is given by Equation (30):

$$(30)\quad \sigma_i^2 = E[(h(Y) - \eta_i)^2 \mid R_i]$$

$$= (\underset{\sim}{M}_2 - \underset{\sim}{M}_1)'\Sigma^{-1}(M_2 - M_1) = 2\eta \quad .$$

Similarly, ε_2 can be obtained by calculating $1 - \Psi(\frac{\eta - t}{\sigma})$, i.e.,

$$(31)\quad \varepsilon_2 = \int_{-\infty}^{t} p(h(Y) \mid R_2)dh(\underset{\sim}{Y}) = 1 - \Psi(\frac{\eta - t}{\sigma}) \quad .$$

Illustration of the model with an example

A 40-item fraction subtraction test was given to 535 students at a local junior high school. A computer program adopting a deterministic strategy for diagnosing erroneous rules of operation in subtracting two fractions was developed on the PLATO system. The students' performances on the test were analyzed by the error-diagnostic program and summarized by Tatsuoka (1984a). In order to illustrate the rule space model and the decision rule described in the previous section, two very common erroneous rules (Tatsuoka, 1984a) are chosen to explain the model.

Rule 8. This rule is applicable to any fraction or mixed number. A student subtracts the smaller from the larger number in unequal corresponding parts and keeps corresponding equal parts as is in the answer. Examples are,

1. 4 4/12 - 2 7/12 = 2 3/12 = 2 1/4
2. 7 3/5 - 4/5 = 7 1/5
3. 3/4 - 3/8 = 3/4

Rule 30. This rule is applicable to the subtraction of mixed numbers where the first numerator is smaller than the second numerator. A student reduces the whole-number part of the minuend by one and adds one to the tens digit of the numerator.

1. 4 4/12 - 2 7/12 = 3 14/12 - 2 7/12 = 1 7/12
2. 3 3/8 - 2 5/6 = 2 13/8 - 2 5/6 = 19/24
3. 7 3/5 - 4/5 = 6 13/5 - 4/5 = 2 9/5

These two rules are applied to 40 items and two sets of responses are scored by "right or wrong" scoring procedure. The binary score pattern made by Rule 8 is denoted by $\underset{\sim}{R}_8$ and the other made by Rule 30 is denoted by $\underset{\sim}{R}_{30}$.

Besides the two rule mentioned above, 38 different error types are identified by a task analysis. However, these error types do not necessarily represent microlevels of cognitive processes such as erroneous rules of operaton. They are somehow, defined more coarsely, like borrowing errors are grouped as a single error type, or the combination of borrowing and getting the least common multiple of two denominators is counted as one error type. In other words, 38 binary reponse patterns representing 38 error types are obtained.

The 535 students' responses on the 40 items are scored and used for estimating item parameters a_j and b_j by the maximum likelihood procedure. By using these a- and b-values, θ-values associated with the two rules and 38 error types are computed. Then corresponding ξ-values are calculated. Thus, 40 points, $(\hat{\theta}_k, \xi_k)$, k=1,...,40 are plotted in the rule space (Rule 8 is renumbered to 39 and Rule 30 to 40. It is only coincidence that the number of rules equals the number.

Insert Table 1 about here

Table 1

The 40 Centroids Representing 40 different error types in Fraction Subtraction Tests (N = 535, n = 40)

Group	θ	ζ	No. of Items	Group	θ	ζ	No. of Items
1	-2.69	-.80	1	21	.24	-.89	22
2	-1.22	-.69	4	22	-.22	-1.23	14
3	-.75	-.68	8	23	.62	-1.55	32
4	-.46	.75	10	24	1.04	-.61	38
5	.11	.91	18	25	.75	-.05	34
6	.64	1.74	30	26	-.51	-1.62	10
7	-.17	1.48	13	27	-.87	-.56	6
8	.40	-.16	25	28	-1.99	1.01	2
9	.60	-.43	31	29	-.19	1.53	12
10	.57	-.24	29	30	-.24	2.74	10
11	.99	.72	37	31	-1.18	1.46	4
12	1.19	.86	39	32	-1.45	.58	4
13	-.60	-1.58	10	33	.64	1.74	30
14	-.44	-2.31	12	34	.57	-.66	31
15	-.18	.67	14	35	.59	-1.39	30
16	-.08	-1.81	16	36	-1.66	-1.96	4
17	.16	-.86	20	37	-.52	-.94	10
18	-.01	-2.12	18	38	-.32	-1.26	14
19	.09	-2.26	20	39	-.41	-2.57	13
20	.29	-1.51	24	40	.17	-2.34	22

*These items will have the score of 1, otherwise the score will be 0.

Now, two students A and B who used Rules 8 and 30 for a subset of 40 items are selected. This was possible because their performances are diagnosed independently by the error-diagnostic system SPFBUG mentioned in Tatsuoka (1984b). The circles shown in Figure 3 represent A and B. Their Mahalanobis distances, D^2, to the 40 centroids are calculated respectively and the smallest values of two distances, D^2 , are selected to compute probabilities of errors. Table 2 summarizes the results.

Insert Table 2 & Figure 3 about here

The D^2 values of Student A to Sets 40 and 19 are 0.008 and 0.119, respectively, and both the values are small enough to judge that A may be classified to either of the sets. Since D^2 follows the χ^2-distribution with two degrees of freedom (Tatsuoka, 1971), the null hypotheses that $D^2_{(A,\text{Set 40})} = 0$ and $D^2_{(A,\text{Set 19})} = 0$ cannot be rejected at, say $\alpha = .25$. The error probabilities ε_1 and ε_2 are .581, .266, respectively. Therefore, we conclude A belongs to Set 19 although $D^2_{(A,\text{Set 40})}$ is smaller than $D^2_{(A,\text{Set 19})}$. This happened because the prior probability of Prob(Set 40) is smaller than that of Prob(Set 19), where the threshold value, t, is determined as follows:

$$t = -\ell n \left[\text{Prob(Set 40)} \,/\, \text{Prob(Set 19)} \right]$$

and $$\text{Prob(Set k)} \propto (1/2\pi) \exp\left[-(\hat{\theta}_k, \xi_k)' \, \Sigma_k^{-1} \, (\hat{\theta}_k, \xi_k)/2 \right] .$$

Table 2

Summary of Classification Results of Students A and B

	Student A		Student B	
D^2	$D^2_{A,\ Set\ 40}$	.008	$D^2_{B,\ Set\ 39}$	.021
	$D^2_{A,\ Set\ 19}$	.119	$D^2_{B,\ Set\ 14}$	.135
ε_1	.581		.979	
ε_2	.266		.010	
η	.088		.040	
t	-.174		-.613	

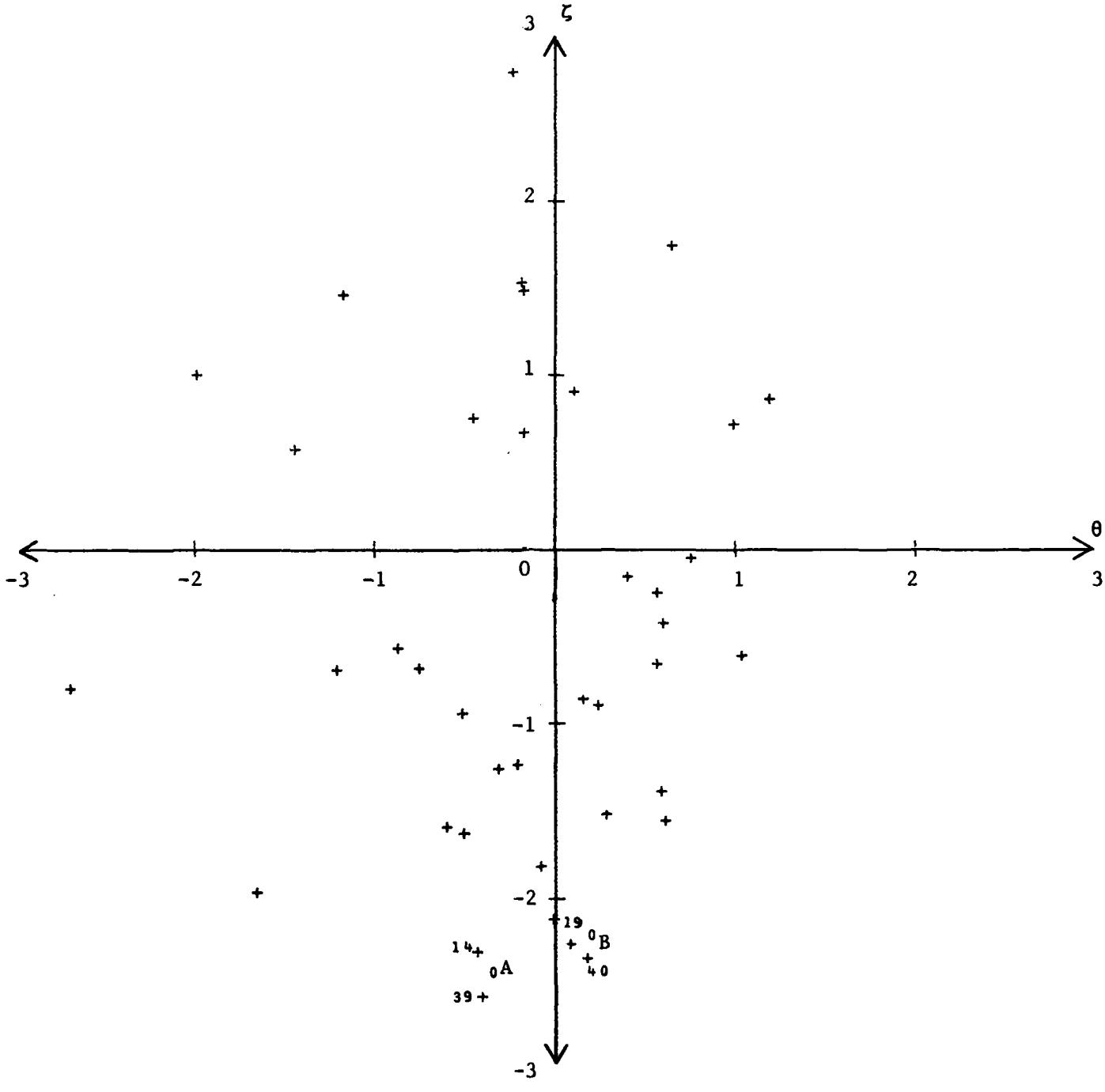

Figure 3: Forty Centroids (+) Representing Different Error Types Determined by a Detailed Task Analysis and Students A and B ($_0$).

Discussion

A new probabilistic model that is capable of measuring cognitive-skill acquisition, and of diagnosing erroneous rules of operation in a procedural domain was introduced by Tatsuoka and her associates (Tatsuoka, 1985; Tatsuoka & Baillie, 1982; Tatsuoka & Tatsuoka, 1982; Tatsuoka, 1983; Tatsuoka, 1984a). The model, called rule space, involves two important components: 1) determination of a set of bug distributions, or in other words, bug density functions representing clusters around the rules, and 2) establishment of decision rules for classifying an observed response pattern into one of the clusters around the rules and computing error probabilities. If each cluster around a rule can be described by a bivariate normal distribution of θ and ζ, then application of the techniques available in the theory of statistical classification and pattern recognition is fairly straightforward and easy.

This study introduces the fact that the cluster around the rule consisting of the response patterns resulting from one, two,..., several slips away from perfect applicaton of the rule indeed follows a compound binomial distribution with centroid (θ_R, ζ_R) and variance $\sum_{j=1}^{n} p_j q_j$, where p_j $(j=1,...,n$ is the probability of having a slip from Rule R for item j. The values of p_j and q_j are approximated by the logistic probabilities $P_j(\theta_R)$ and $Q_j(\theta_R)$, $j=1,...,n$, in this study instead of estimating them from the dataset. Plausibility of the approximation of the slip probabilities associated with each erroneous rule by the logistic function is left as a future topic of investigation, although the fit with data seems to be good.

The determination of a set of ellipses representing clusters around the rules can be automatic after all the erroneous rules are discovered. Many researchers in cognitive science and artificial intelligence have started constructing error diagnostic systems in various domains in this decade. Expert teachers usually know their students' errors, as well as the weaknesses and strengths of each child's knowledge structure. Since the model does not require a large-scale computation such as strategies commonly used in the area of artifical intelligence do, the rule-space model is helpful in more general areas of research and teaching, and for those who have microcomputers for testing their hypotheses, validating their data with probabilistically-sound information, and evaluating their teaching methods and materials. Moreover, the model can be "intelligent" in the sense that the researcher can improve and modify the information for the cluster ellipses as they get more new students whose performances they can study.

The set of ellipses can represent many things besides erroneous rules. They can represent specific contents of some domain, usage errors in the language arts, or processes required in algebra. However, further research is necessary to develop methods for determining the set of ellipses other than relying on an expert teacher. The method must be efficient and compatible with the recent theories of human cognition and learning.

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